MATH-120 Practice Midterm

You have 50 min. to solve 6 problems. Show your work; substantiate your solutions where necessary. Calculators, notes, formula sheets are not allowed on this exam.

1. In each of the following cases evaluate the expression or briefly explain why it is not defined.

a) $\begin{bmatrix} 1 & 8 & 0 \\ -2 & 5 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 6 \end{bmatrix}$

b) $\left(\begin{bmatrix} -1 & 2 & 5 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \right) \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 4 \end{bmatrix}$

2. Find all the values of the parameters a and b such that the following linear system (i) is inconsistent; (ii) has infinitely many solutions; (iii) has exactly one solution.

$$\begin{aligned} 2x - 7y + 3z &= b \\ x - 5y + 4z &= -3 \\ -2x + y + az &= 0 \end{aligned}$$

3. In this problem you should use properties of determinants. Credit will not be given if the properties are not used.

a) Evaluate the following determinant: $\begin{vmatrix} -2 & 0 & 3 & 1 \\ -1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 3 \\ 1 & 2 & 2 & 4 \end{vmatrix}$

b) Prove that

$$\begin{vmatrix} a_1 + b_1 & a_1 - b_1 & 1 \\ a_2 + b_2 & a_2 - b_2 & 1 \\ a_3 + b_3 & a_3 - b_3 & 1 \end{vmatrix} = -2 \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

without evaluating the determinants.

4. a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ if it exists.

b) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -8 \\ 2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & 2 \\ 1 & -2 \end{bmatrix}$. One can check (you don't have to) that $AB = AC$ although $B \neq C$. Explain, why this is possible.

5. For each of the following statements decide whether it is true (T) or false (F). You do not have to substantiate your decision.

a) A system is consistent if the number of equations does not exceed the number of unknowns (variables) of the system.

b) If the coefficient matrix of a homogeneous system with 5 equations for 5 unknowns is invertible, the system will have only a trivial solution.

c) If A and B are square matrices of the same size, then
det(A+B) = detA + detB

d) If A is a $k \times k$ matrix and detA = 3, then det(2A) = 6.

e) $\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 2a+1 & 2b+2 & 2c+3 \\ d & e & f \\ 1 & 2 & 3 \end{vmatrix}$

6. Use Cramer's rule to find y from the following system:

$$\begin{aligned} 2x - y + z + t &= 9 \\ x + 3y \quad - t &= 2 \\ 3x + y + z \quad &= 9 \\ -x - y - z + 2t &= 3 \end{aligned}$$